

# $\Lambda(t)$ CDM Model as a Unified Origin of Holographic and Agegraphic Dark Energy Models

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Motivated by the fact that any nonzero  $\Lambda$  can introduce a length scale or a time scale into Einstein's theory,  $r_\Lambda = ct_\Lambda = \sqrt{3/|\Lambda|}$ . Conversely, any cosmological length scale or time scale can introduce a  $\Lambda(t)$ ,  $\Lambda(t) = 3/r_\Lambda^2(t) = 3/(c^2 t_\Lambda^2(t))$ . In this letter, we investigate the time varying  $\Lambda(t)$  corresponding to the length scales, including the Hubble horizon, the particle horizon and the future event horizon, and the time scales, including the age of the universe and the conformal time. It is found out that, in this scenario, the  $\Lambda(t)$ CDM model can be taken as the unified origin of the holographic and agegraphic dark energy models with interaction between the matter and the dark energy, where the interacting term is determined by  $Q = -\dot{\rho}_\Lambda$ . We place observational constraints on the  $\Lambda(t)$ CDM models originating from different cosmological length scales and time scales with the recently compiled “Union2 compilation” which consists of 557 Type Ia supernovae (SNIa) covering a redshift range  $0.015 \leq z \leq 1.4$ . In conclusion, an accelerating expansion universe can be derived in the cases taking the Hubble horizon, the future event horizon, the age of the universe and the conformal time as the length scale or the time scale.

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## I. INTRODUCTION

The accelerating expansion of the universe is one of the most important issues of modern cosmology, which has been discovered and verified by supernova [1], CMB [2] and BAO [3] observations (see also recent reviews [4, 5]). After the discovery of this scenario, a great variety of attempts have been done to explain this acceleration (see the reviews [5, 6]). Among these alternatives,  $\Lambda$ CDM is the simplest and most nature one, which fits the observational data best. In this scenario, the dark energy is associated to the energy density of the quantum vacuum. Briefly put, the dark energy is the energy stored on the true vacuum state of all existing fields in the universe, i.e.  $\rho_\Lambda = \Lambda/8\pi G$ , where  $\Lambda$  is the cosmological constant. However, it is embarrassed by cosmological constant problems, namely the fine-tuning problem and the coincidence problem [7]. Several possible approaches have been adopted to explain or alleviate the cosmological constant problems [8], though there is no convincing fundamental theory for why vacuum energy dominance happened only recently. The possibility that if  $\Lambda$  is not a real constant but is time variable or dynamical, the cosmological problems may be alleviated or removed, was considered many years ago [9, 10]. This kind of models can be called as  $\Lambda(t)$ CDM models. The traditional approach for  $\Lambda(t)$ CDM models was first to specify a phenomenological time-dependence form for  $\Lambda(t)$  and then establish a cosmological scenario. There are a lot of proposals of the phenomenological forms for  $\Lambda(t)$  in the literature [11], such as  $\rho_\Lambda = \sigma H$  and  $\rho_\Lambda = n_1 H + n_2 H^2$ . In our scenario,  $\Lambda(t)$  is determined based on its relation with the cosmological length scale or time scale.

As it is known, in the de Sitter universe, any nonzero  $\Lambda$  can introduce a length scale  $r_\Lambda$  and time scale  $t_\Lambda$ , as the form

$$r_\Lambda = ct_\Lambda = \sqrt{3/|\Lambda|}, \quad (1)$$

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into Einstein's theory [12], where  $c$  is the speed of light and throughout this letter we use the unit  $c = 1$ . Conversely, a cosmological length scale and time scale may introduce a  $\Lambda(t)$  into Einstein's theory

$$\Lambda(t) = \frac{3}{r_\Lambda^2(t)} = \frac{3}{t_\Lambda^2(t)}. \quad (2)$$

Obviously, when the length scale or time scale is time variable, a time varying  $\Lambda$  can be obtained. The key problem is how to choose a proper cosmological length scale or time scale to obtain a tiny  $\Lambda$ . However, we have not acquired the first physical principle to determine the length or time scale recently. Nevertheless, one can immediately relate these length or time scales to the biggest natural length scales, including the Hubble horizon, the particle horizon and the future event horizon, and natural time scales, including the age of the universe and the conformal time. The cosmological constant derived from the length scale can be called as *horizon cosmological constant* [13], and the one derived from the time scale can be called as *age cosmological constant* [14].

Once mentioning the length scale and time scale, one may easily associate them with the holographic and agegraphic dark energy. Based on the holographic principle, Cohen et al. [15] suggested a relation between the IR cut-off and the UV cut-off in the quantum field theory. For a system with size  $L$  representing the IR cut-off and UV cut-off  $\Lambda$  relating to the quantum zero-point energy, without decaying into a black hole of the same size, it is required that the total energy in a region of size  $L$  should not exceed the mass of a black hole of the same size, namely  $L^3 \rho_\Lambda \leq LM_P^2$ . When this inequality is reduced to an equality

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2}, \quad (3)$$

$L$  takes the maximum value, where  $M_P \equiv 1/\sqrt{8\pi G}$  is the reduced Planck mass and  $3c^2$  is a numerical factor. Recently, three length scales of the universe: the Hubble horizon, the particle horizon and the future event horizon, have been taken as the role of IR cut-off. However, when the Hubble horizon and the particle horizon are chosen as the IR cut-off, non-accelerated expansion universe can be achieved. Li [16] proposed to take the future event horizon as the IR cut-off leading to the holographic dark energy model which can derive an accelerated expansion universe. Also, this model has been constrained with different observational data and is consistent with the data [17]. However, it is embarrassed on the fundamental level due to its assumption that the current properties of the dark energy are determined by the future evolution of the universe, which is the so-called causality problem [18]. What's more, it has been pointed out that this model can be inconsistent with the age of the universe [19]. Recently, based on the Károlyházy uncertainty relation  $\delta t = \beta t_p^{2/3} t^{1/3}$  together with the time-energy uncertainty relation, also called the Heisenberg uncertainty relation  $E_{\delta t^3} \sim t^{-1}$ , one can estimate the quantum energy density of the metric fluctuations of Minkowski space-time that is  $\rho_\Lambda = 3n^2 M_P^2/t^2$ , where  $3n^2$  is a numerical factor representing some uncertainties. In the above context,  $\beta$  is a numerical factor of order one,  $t_p$  is the reduced Planck time, and throughout this letter, the units  $c = \hbar = k_b = 1$  are adopted, so that one has  $l_p = t_p = 1/m_p$  with  $l_p$  and  $m_p$  being the reduced Planck length and mass, respectively. Recently, two time scales of the universe: the age of the universe and the conformal time, have been taken as the role of  $t$ , corresponding to the agegraphic dark energy [18] and the new agegraphic dark energy [20]. Both of the models can derive an accelerated expansion universe, and is consistent with the recent observational data [21]. They two also can resolve the causality problem, however, it has been presented that the agegraphic dark energy model is classically unstable, and the new agegraphic dark energy model is no better than the holographic dark energy model for the description of the dark energy-dominated universe [22]. After a brief introduction of the holographic and agegraphic dark energy, one may be more interested in that whether there are some relationships between them and the  $\Lambda(t)$ CDM models investigated in this letter. The corresponding problems will be discussed in Sections III and IV.

The letter is organized as follows. In Section II, the basic equations of the time variable cosmological constant models are given. In Section III, three length scales and two time scales will be considered to derive the time variable cosmological constant models, and constraints from the recent observational data are illustrated. Furthermore, the relationships between the time variable cosmological constant models and the holographic dark energy and agegraphic dark energy are investigated. Finally, we provide our conclusion and discussion in Section IV.

## II. $\Lambda(t)$ CDM COSMOLOGY: BASIC EQUATIONS

Considering the Einstein field equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu}, \quad (4)$$

where  $T_{\mu\nu}$  is the energy-momentum tensor of matter and radiation, one sees by Bianchi identities that when the energy-momentum tensor is conserved, namely  $\nabla^\mu T_{\mu\nu} = 0$ , it follows necessarily that  $\Lambda$  is a constant. To accommodate the

running of  $\Lambda$  with the cosmic time, namely  $\Lambda = \Lambda(t)$ , the most obvious way is to shift  $\Lambda(t)$  to the right-hand side of Eq. (4) and to take  $\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{\Lambda(t)}{8\pi G}g_{\mu\nu}$  as the total energy-momentum tensor. By requiring the local energy-momentum conservation law,  $\nabla^\mu \tilde{T}_{\mu\nu} = 0$ , one yields

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \quad (5)$$

with  $H = \dot{a}/a$  being the Hubble parameter, where an over dot means taking derivative with respect to the cosmic time  $t$ . In a spacially flat FRW universe ignoring the radiation, with  $\rho_T = \rho_m + \rho_\Lambda$  and  $p_T = p_m + p_\Lambda$ , the Friedmann equation can be written as

$$H^2 = \frac{1}{3M_P^2}(\rho_m + \rho_\Lambda). \quad (6)$$

With the definitions of the dimensionless density parameters  $\Omega_m = \rho_m/3M_P^2H^2$  and  $\Omega_\Lambda = \rho_\Lambda/3M_P^2H^2$ , one can reduce Eq. (6) to

$$\Omega_\Lambda + \Omega_m = 1. \quad (7)$$

In this situation, Eq. (5) is reduced to

$$\dot{\rho}_\Lambda + \dot{\rho}_m + 3H(1 + w_m)\rho_m + 3H(1 + w_\Lambda)\rho_\Lambda = 0. \quad (8)$$

Throughout, the subscript “ $m$ ” denotes the corresponding quantity of the matter which contains the baryonic and dark matter, as well as, “ $\Lambda$ ” represents the corresponding quantity of the vacuum energy and “ $T$ ” indicates the corresponding quantity of total components.  $w_i = p_i/\rho_i$  is the equation of state (EoS) of the  $i$ th-component (here  $i = m, \Lambda$ ), where  $\rho_i$  and  $p_i$  are density and pressure of corresponding component, with  $\rho_\Lambda = M_P^2\Lambda(t)$ .

Assuming the vacuum energy and matter exchange energy through interaction term  $Q$ , then Eq. (8) can be rewritten as

$$\dot{\rho}_m + 3H(1 + w_m)\rho_m = Q, \quad (9)$$

and

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = -Q. \quad (10)$$

Rewriting Eqs. (9) and (10) as

$$\dot{\rho}_m + 3H(1 + w_m^{eff})\rho_m = 0 \quad (11)$$

and

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda^{eff})\rho_\Lambda = 0, \quad (12)$$

one can define the effective EoS for the matter and the vacuum energy as

$$w_m^{eff} = w_m - \frac{Q}{3H\rho_m}, \quad (13)$$

and

$$w_\Lambda^{eff} = w_\Lambda + \frac{Q}{3H\rho_\Lambda}. \quad (14)$$

For the pressureless matter, one reads  $p_m = 0$  and  $w_m = 0$ . Within this framework, it is interesting to mention that the EoS of the vacuum energy takes the usual form  $w_\Lambda(t) = p_\Lambda(t)/\rho_\Lambda(t) = -1$  [10]. It shows that the value of this EoS does not depend on whether  $\Lambda$  is strictly constant or time variable. Furthermore, Eqs. (9) and (10) are reduced as

$$\dot{\rho}_\Lambda = -Q \quad (15)$$

and

$$\dot{\rho}_m + 3H\rho_m = -\dot{\rho}_\Lambda, \quad (16)$$

which shows that the matter and the vacuum energy are not independently conserved, with the decaying vacuum density  $\rho_\Lambda$  playing the role of a source of matter production. In this regard, the  $\Lambda(t)$ CDM model also can be called as the decaying vacuum model. Jointing Eqs. (13), (14) and (15), one obtains

$$\begin{aligned} w_\Lambda^{eff} &= -1 - \frac{\dot{\rho}_\Lambda}{3H\rho_\Lambda} \\ &= -1 - \frac{1}{3} \frac{d \ln \rho_\Lambda}{d \ln a} \end{aligned} \quad (17)$$

and

$$\begin{aligned} w_m^{eff} &= -(w_\Lambda^{eff} + 1) \frac{\rho_\Lambda}{\rho_m} \\ &= -(w_\Lambda^{eff} + 1) \frac{\Omega_\Lambda}{1 - \Omega_\Lambda}, \end{aligned} \quad (18)$$

Using Eqs. (6) and (11), one gets the following solution

$$E^2(z) = \frac{(1 - \Omega_{\Lambda 0}) \exp\{3 \int_0^z \frac{dz'}{1+z'} [1 + w_m^{eff}(z')]\}}{1 - \Omega_\Lambda}, \quad (19)$$

where  $E = H/H_0$  is the dimensionless Hubble parameter. Throughout the subscript “0” denotes the value of a quantity today. Moreover, substituting Eq. (6) into Eq. (8), one yields

$$\frac{d \ln H}{d \ln a} + \frac{3}{2}(1 - \Omega_\Lambda) = 0. \quad (20)$$

According to the definitions of the decelerating parameter  $q \equiv -(\ddot{a}a)/\dot{a}^2$  and the Hubble parameter  $H \equiv \dot{a}/a$ , one can obtain

$$q = (-\frac{\ddot{a}}{a})/H^2 = -1 - \frac{d \ln H}{d \ln a}. \quad (21)$$

Furthermore, from Eqs.(20) and (21), one reads

$$q = \frac{1}{2} - \frac{3\Omega_\Lambda}{2}, \quad (22)$$

which shows that to ensure the current accelerating expansion of the universe,  $\Omega_{\Lambda 0} > 1/3$  is required. With Eq.(17) and the definition of the dimensionless energy density  $\Omega_\Lambda = \rho_\Lambda/3M_p^2 H^2$ , the effective EoS of the vacuum energy is given by

$$w_\Lambda^{eff} = -1 - \frac{1}{3} \left( \frac{d \ln \Omega_\Lambda}{d \ln a} + 2 \frac{d \ln H}{d \ln a} \right). \quad (23)$$

Moreover, with Eqs.(20) and (23), one obtains

$$w_\Lambda^{eff} = -\Omega_\Lambda - \frac{1}{3} \frac{d \ln \Omega_\Lambda}{d \ln a}. \quad (24)$$

Substituting Eq. (24) into Eq. (18), one gets

$$w_m^{eff} = \frac{\Omega_\Lambda}{1 - \Omega_\Lambda} \left( \Omega_\Lambda + \frac{1}{3} \frac{d \ln \Omega_\Lambda}{d \ln a} - 1 \right). \quad (25)$$

### III. CONSTRAINTS FROM THE RECENT OBSERVATIONAL DATA

#### A. Horizon cosmological constant

##### 1. Hubble horizon as a cosmological length scale

When Hubble horizon  $H^{-1}$  is chosen, one obtains

$$\Lambda(t) = 3c^2 H^2(t) \quad (26)$$

where  $c$  is a constant. As it is known, our universe is filled with the matter and dark energy which deviates from a de Sitter universe. Just to describe this gap, the constant  $c$  was introduced in [13]. Naturally, when  $c = 1$ , the de Sitter universe will be recovered. Now, according to the definition of the energy density  $\rho_\Lambda = M_P^2 \Lambda(t)$ , the corresponding vacuum energy density can be written as

$$\rho_\Lambda = 3c^2 M_P^2 H^2 \quad (27)$$

which takes the same form as the holographic dark energy with the Hubble horizon based on holographic principle [16]. With this vacuum energy, the Friedmann equation (6) can be rewritten as

$$\rho_m = 3(1 - c^2) M_P^2 H^2. \quad (28)$$

Requiring a positive value for the matter energy density  $\rho_m$ , the condition

$$c^2 < 1, \quad (29)$$

must be satisfied. According to the definition of the dimensionless energy density, one has

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{3c^2 M_P^2 H^2}{3M_P^2 H^2} = c^2. \quad (30)$$

In this case, from Eqs.(30) and (22), the decelerating parameter becomes

$$q = \frac{1}{2} - \frac{3}{2}c^2. \quad (31)$$

To obtain a current accelerating expansion universe, i.e.  $q < 0$ , and to ensure a positive matter energy density, the condition

$$1/3 < c^2 < 1 \quad (32)$$

is necessary. Furthermore, from Eqs.(30) and (24), the effective EoS of vacuum energy density is

$$w_\Lambda^{eff} = -c^2. \quad (33)$$

With the condition Eq. (32), one can see that  $-1 < w_\Lambda^{eff} < -1/3$ , which means that a quintessence-like dark energy is obtained. Substituting Eq. (30) into Eq. (25), one read

$$w_m^{eff} = -c^2. \quad (34)$$

With Eqs. (19), (30) and (34), the Friedmann equation of this model is reduced to

$$E^2(z; c) = (1 + z)^{3(1-c^2)}, \quad (35)$$

which shows that there is only one model-dependent parameter  $c$ .

We perform the  $\chi^2$  statistics to constrain the parameter  $c$  in this model with the recently compiled ‘‘Union2 compilation’’ which consists of 557 Type Ia supernovae (SNIa) covering a redshift range  $0.015 \leq z \leq 1.4$  [23]. The best-fit value for parameter  $c$  is  $c_b = 0.766$  with  $\chi_{min}^2 = 550.767$ . The confidence range is  $0.750 \leq c \leq 0.781$ , i.e.  $c = 0.766_{-0.016}^{+0.015}$  with 68.3% confidence level. Furthermore, one can work out  $\Omega_{m0} = 0.413_{-0.023}^{+0.025}$  and  $\Omega_{\Lambda0} = 0.587_{-0.025}^{+0.023}$  with 68.3% confidence level. Fig. 1(a) shows the evolution of  $\chi^2$  with respect to the parameter  $c$ . Fig. 1(b) displays the probability distribution of  $c$ , where the probability is defined as  $p = \exp(-\chi^2/2)/\exp(-\chi_{min}^2/2)$ .

## 2. Particle horizon as a cosmological length scale

The particle horizon is defined as

$$R_p = a(t) \int_0^t \frac{dt'}{a} = a \int_0^a \frac{da'}{Ha'^2} \quad (36)$$

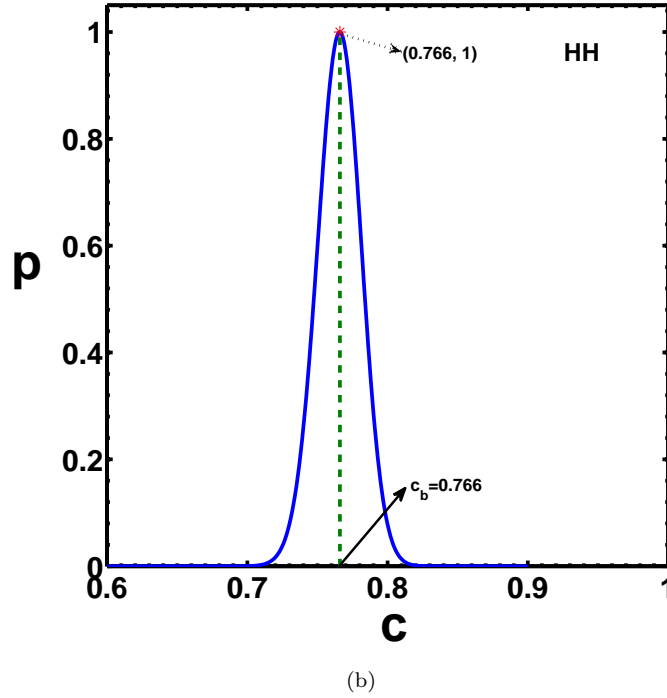
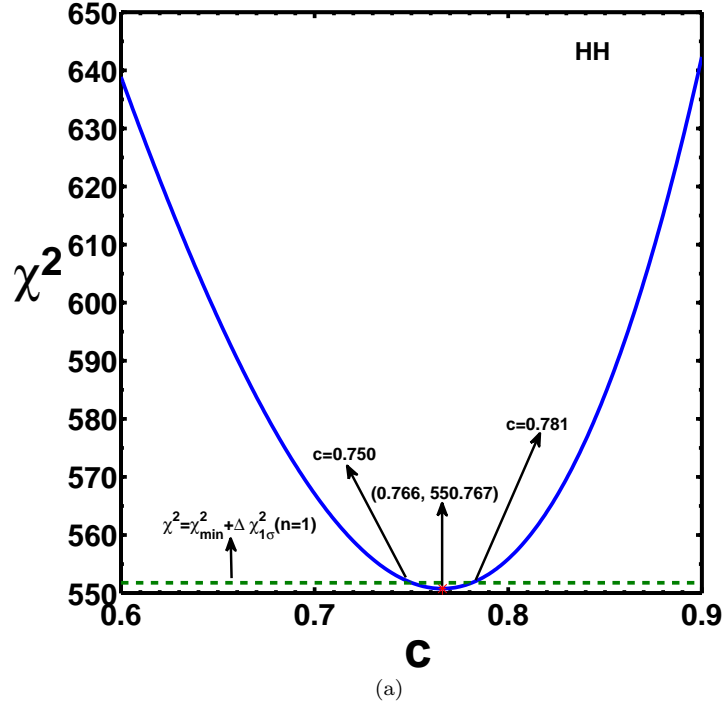


FIG. 1: Hubble horizon is taken as a cosmological length scale (HH for short). The best-fit value for parameter  $c$  is  $c_b = 0.766$  with  $\chi^2_{min} = 550.767$ . The confidence range is  $0.750 \leq c \leq 0.781$ , i.e.  $c = 0.766^{+0.015}_{-0.016}$  with 68.3% confidence level. Fig. 1(a) shows the evolution of  $\chi^2$  with respect to the parameter  $c$ . The green dashed line denotes  $\chi^2 = \chi^2_{min} + \Delta\chi^2_{1\sigma}(n = 1)$ . The red dot indicates the best-fit pair  $(c, \chi^2_{min}) = (0.766, 550.767)$ . Fig. 1(b) displays the probability distribution of  $c$ , where the probability is defined as  $p = \exp(-\chi^2/2)/\exp(-\chi^2_{min}/2)$ . The green dashed line denotes  $c = c_b$ .

which is the length scale a particle can pass from the beginning of the universe  $t = 0$ . When the particle horizon  $R_p$  is chosen, one gets

$$\Lambda(t) = 3c^2/R_p^2 \quad (37)$$

In this case, the vacuum energy density is given as

$$\rho_\Lambda = 3c^2 M_P^2/R_p^2, \quad (38)$$

which takes the same form as the holographic dark energy with the particle horizon[16], where  $c$  is taken to fill the deviation from the de Sitter universe. Combing Eqs.(36), (38) and the definition of the dimensionless energy density  $\Omega_\Lambda$ , one has

$$\int_0^a \frac{d \ln a'}{H a'} = \frac{c}{aH} \sqrt{\frac{1}{\Omega_\Lambda}}. \quad (39)$$

Taking the derivative with respect  $\ln a$  from the both sides of the above Eq.(39), one has the differential equation

$$\frac{d \ln H}{d \ln a} + \frac{1}{2\Omega_\Lambda} \frac{d \Omega_\Lambda}{d \ln a} = -\frac{\sqrt{\Omega_\Lambda}}{c} - 1 \quad (40)$$

Substituting Eq.(20) into the above Eq.(40), one obtains the differential equation of  $\Omega_\Lambda$

$$\frac{d \Omega_\Lambda}{dz} = -\Omega_\Lambda \left(1 - 3\Omega_\Lambda - \frac{2}{c} \sqrt{\Omega_\Lambda}\right) (1+z)^{-1}. \quad (41)$$

Once the values of  $c$  and  $\Omega_{\Lambda 0}$  are fixed, the evolution of  $\Omega_\Lambda$  as a function of the redshift  $z$  can be obtained. From Eqs.(24) and (41), it is easy to get

$$w_\Lambda^{eff} = -\frac{1}{3} + \frac{2}{3c} \sqrt{\Omega_\Lambda}. \quad (42)$$

Obviously,  $w_\Lambda^{eff} > -1/3$ , hereby, the particle horizon could not describe the accelerating phase.

### 3. Future event horizon as a cosmological length scale

The future event horizon is defined as

$$R_e = a \int_t^\infty \frac{dt'}{a} = a \int_a^\infty \frac{da'}{H a'^2} \quad (43)$$

which is the boundary of the volume a fixed observer may eventually observe[16]. Taking it as the role of cosmological length scale, one has the vacuum energy density

$$\rho_\Lambda = 3c^2 M_P^2/R_e^2, \quad (44)$$

which takes the same form as the holographic dark energy with the future event horizon[16], where the constant  $c$  is also taken to fill the deviation from the de Sitter universe. Combining Eq. (43), Eq. (44) and the definition of the dimensionless energy density  $\Omega_\Lambda$ , one has

$$\int_a^\infty \frac{d \ln a'}{H a'} = \frac{c}{aH} \sqrt{\frac{1}{\Omega_\Lambda}}. \quad (45)$$

Taking the derivative with respect to  $\ln a$  from the both sides of the above equation (45), one has the differential equation

$$\frac{d \ln H}{d \ln a} + \frac{1}{2} \frac{d \ln \Omega_\Lambda}{d \ln a} = \frac{\sqrt{\Omega_\Lambda}}{c} - 1. \quad (46)$$

Substituting Eq. (20) into the above differential equation, one obtains

$$\frac{d \Omega_\Lambda}{dz} = -\Omega_\Lambda \left(1 - 3\Omega_\Lambda + \frac{2}{c} \sqrt{\Omega_\Lambda}\right) (1+z)^{-1} \quad (47)$$

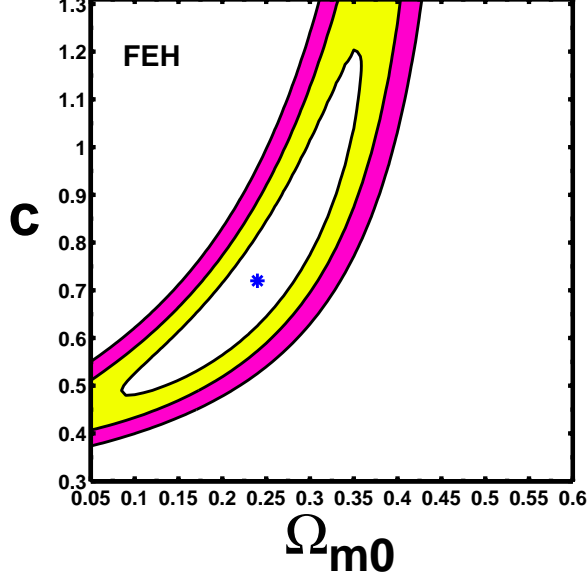


FIG. 2: Future event horizon is taken as a cosmological length scale (FEH for short). The contours correspond to 68.3%, 95.4% and 99.7% confidence levels constrained from the recent observational data in  $(\Omega_{m0}, c)$  plane. The blue dot marks the best-fit pair  $(\Omega_{m0}, c)$ . The best-fit parameters in this case are found to be  $\Omega_{m0} = 0.24^{+0.11}_{-0.16}$  and  $c = 0.72^{+0.49}_{-0.24}$  (68.3% c.l.) with  $\chi^2_{min} = 544.367$ .

With Eqs.(24) and (47), one reads

$$w_{\Lambda}^{eff} = -\frac{1}{3} - \frac{2}{3c}\sqrt{\Omega_{\Lambda}}. \quad (48)$$

Obviously,  $w_{\Lambda}^{eff} < -1/3$ , hence, the future event horizon can lead to the accelerating phase. The decelerating parameter  $q(z)$  is expressed as Eq.(22), where  $\Omega_{\Lambda}$  is determined by Eq. (47). Substituting Eq. (47) into Eq. (25), one reads

$$w_m^{eff} = -\frac{2}{3}\left(1 - \frac{\sqrt{\Omega_{\Lambda}}}{c}\right)\frac{\Omega_{\Lambda}}{1 - \Omega_{\Lambda}}. \quad (49)$$

The Friedmann equation of this model is determined by jointing Eqs. (19), (47) and (49).

We perform the  $\chi^2$  statistics to constrain the parameters  $(\Omega_{m0}, c)$  with the recently compiled “Union2 compilation” of SNeIa data. Figure 2 shows the probability contours constrained from the observational data in  $(\Omega_{m0}, c)$  plane. The best-fit parameters in this case are found to be  $\Omega_{m0} = 0.24^{+0.11}_{-0.16}$  and  $c = 0.72^{+0.49}_{-0.24}$  for 68.3% confidence level with  $\chi^2_{min} = 544.367$ .

## B. Age cosmological constant

### 1. Age of the universe as a cosmological time scale

The age of the universe is defined as

$$t_{\Lambda} = \int_0^t dt' = \int_0^a \frac{da'}{a'H}. \quad (50)$$

Taking it as the role of time scale, one obtains

$$\Lambda(t) = 3c^2/t_{\Lambda}^2, \quad (51)$$



where  $c$  is the model constant to fill the derivation from the de Sitter universe[14]. In this case, one has the vacuum energy density

$$\rho_\Lambda = 3c^2 M_P^2 / t_\Lambda^2, \quad (52)$$

which takes the same form as the agegraphic dark energy[18]. Combining Eq. (50), Eq. (52) and the definition of the dimensionless energy density  $\Omega_\Lambda$ , one has

$$\int_0^a \frac{d \ln a'}{H} = \frac{c}{H} \sqrt{\frac{1}{\Omega_\Lambda}}. \quad (53)$$

Taking the derivative with respect to  $\ln a$  from the both sides of Eq.(53), one has

$$\frac{d \ln H}{d \ln a} + \frac{1}{2} \frac{d \ln \Omega_\Lambda}{d \ln a} + \frac{\sqrt{\Omega_\Lambda}}{c} = 0. \quad (54)$$

Substituting Eq. (20) into Eq.(54), one gets

$$\frac{d \Omega_\Lambda}{dz} = -\Omega_\Lambda (3 - 3\Omega_\Lambda - \frac{2}{c} \sqrt{\Omega_\Lambda}) (1+z)^{-1} \quad (55)$$

From Eqs.(24) and (55), one has

$$w_\Lambda^{eff} = -1 + \frac{2}{3c} \sqrt{\Omega_\Lambda}. \quad (56)$$

The accelerating universe requires  $w_\Lambda^{eff} < -1/3$ , that is,  $c > \sqrt{\Omega_\Lambda}$ . With  $0 \leq \Omega_\Lambda \leq 1$ , naturally, if  $c > 1$  the universe will eternally accelerate. For  $c < 1$ , the expansion would finally slow down in the future. With Eqs. (25) and (55), one yields

$$w_m^{eff} = -\frac{2\sqrt{\Omega_\Lambda}}{3c} \frac{\Omega_\Lambda}{1 - \Omega_\Lambda}. \quad (57)$$

Joining Eqs. (19), (55) and (57), one can determine the Friedmann equation of this model.

Using the recently compiled “Union2 compilation” of SNeIa data, We perform the  $\chi^2$  statistics to constrain the parameters  $(\Omega_{m0}, c)$  in this model. The best-fit results are  $\Omega_{m0} = 0.28_{-0.07}^{+0.08}$  and  $c = 23.3_{-21.3}^{+null}$  for 68.3% confidence level with  $\chi_{min}^2 = 544.25$ , where “null” denotes the absence of the upper limit for  $c$ . Figure 3 displays the probability contours constrained from the observational data in  $(\Omega_{m0}, c)$  plane. However the top sides of the the three contours are not closed, which denotes that we cannot get the upper limit for  $c$  in 68.3%, 95.4% and 99.7% confidence levels. As a general illustration, we work out  $\chi_{c=\infty}^2 = 544.42$  with the best-fit value  $\Omega_{m0} = 0.28$ . Obviously,  $\Delta\chi^2 = \chi_{c=\infty}^2 - \chi_{min}^2 = 0.17$  is smaller than  $\Delta\chi_{1\sigma}^2(n=2) = 2.30$  where  $n$  is the number of the parameters in the model, which implies that  $c = \infty$  is still inside the  $1\sigma$  contour. Basically, this problem originates from the evolution of  $\Omega_\Lambda$  with respect to redshift  $z$  with the variety of  $c$ , that will be further discussed in Section IV.

## 2. Conformal time as a cosmological time scale

The conformal time is defined as

$$\eta_\Lambda = \int_0^t \frac{dt'}{a} = \int_0^a \frac{da'}{a'^2 H}, \quad (58)$$

which is the maximum comoving distance to a comoving observer’s particle horizon since  $t = 0$ [24]. In this case, one has

$$\rho_\Lambda = 3c^2 M_P^2 / \eta_\Lambda^2, \quad (59)$$

which takes the same form as the new agegraphic dark energy[20]. Combining Eq. (58), Eq. (59) and the definition of the dimensionless energy density  $\Omega_\Lambda$ , one has

$$\int_0^a \frac{d \ln a'}{a' H} = \frac{c}{H} \sqrt{\frac{1}{\Omega_\Lambda}}. \quad (60)$$

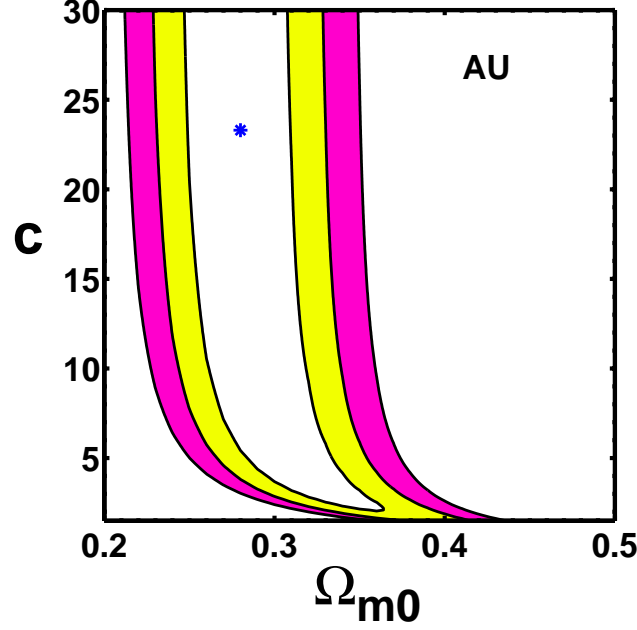


FIG. 3: Age of the universe is taken as a cosmological time scale (AU for short). The contours correspond to 68.3%, 95.4% and 99.7% confidence levels constrained from the recent observational data. The blue dot marks the best-fit pair  $(\Omega_{m0}, c)$ . The results are  $\Omega_{m0} = 0.28^{+0.08}_{-0.07}$  and  $c = 23.3^{+null}_{-21.3}$  (68.3% c.l.) with  $\chi^2_{min} = 544.25$ , where “null” denotes the absence of the upper limit for  $c$ .

Taking the derivative with respect to  $\ln a$  from the both sides of Eq.(60), one gets

$$\frac{d \ln H}{d \ln a} + \frac{1}{2} \frac{d \ln \Omega_{\Lambda}}{d \ln a} + \frac{\sqrt{\Omega_{\Lambda}}}{ac} = 0. \quad (61)$$

With Eqs. (20) and (61), one obtains

$$\frac{d\Omega_{\Lambda}}{dz} = -\Omega_{\Lambda} \left[ \frac{3(1-\Omega_{\Lambda})}{1+z} - \frac{2}{c} \sqrt{\Omega_{\Lambda}} \right] \quad (62)$$

From Eqs.(24) and (62), the equation of state of dark energy is written as

$$w_{\Lambda}^{eff} = -1 + \frac{2}{3c} (1+z) \sqrt{\Omega_{\Lambda}}. \quad (63)$$

The accelerating universe requires  $w_{\Lambda}^{eff} < -1/3$ , that is  $c > \sqrt{\Omega_{\Lambda}}(1+z)$ . Combining Eqs. (25) and (62), one obtains

$$w_m^{eff} = -\frac{2(1+z)\sqrt{\Omega_{\Lambda}}}{3c} \frac{\Omega_{\Lambda}}{1-\Omega_{\Lambda}}. \quad (64)$$

Furthermore, one determine the Friedmann equation of this model via Eqs. (19), (62) and (64).

In Figure 4, we plot the probability contours constrained from the recently compiled “Union2 compilation” of SNeIa data for 68.3%, 95.4% and 99.7% confidence levels. The fitting results are  $\Omega_{m0} = 0.28^{+0.07}_{-0.03}$  and  $c = 28.3^{+null}_{-25.5}$  for 68.3% confidence level with  $\chi^2_{min} = 544.268$ , where “null” denotes the absence of the upper limit for  $c$ . We can see that the top sides of the three contours also are open as the above model. Repeating the analysis and calculations as done in the above case,  $\Delta\chi^2 = \chi^2_{c=\infty} - \chi^2_{min} = 0.151$  is also smaller than  $\Delta\chi^2_{1\sigma}(n=2) = 2.30$ . The further discussion also will be shown in Section IV.

#### IV. DISCUSSION AND CONCLUSION

In this letter, motivated by the fact that any nonzero  $\Lambda$  can introduce a length scale or a time scale into Einstein’s theory,  $r_{\Lambda} = ct_{\Lambda} = \sqrt{3/|\Lambda|}$ . Conversely, any cosmological length scale or time scale can introduce a  $\Lambda(t)$ ,  $\Lambda(t) =$

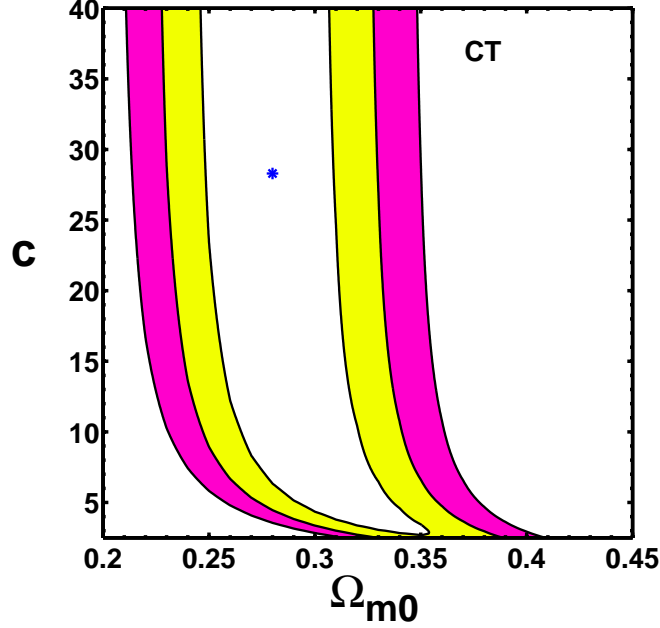


FIG. 4: Conformal time is taken as a cosmological time scale (CT for short). The contours correspond to 68.3%, 95.4% and 99.7% confidence levels constrained from the recent observational data. The blue dot marks the best-fit pair  $(\Omega_{m0}, c)$ . The fitting values are  $\Omega_{m0} = 0.28^{+0.07}_{-0.03}$  and  $c = 28.3^{+null}_{-25.5}$  (68.3% c.l.) with  $\chi^2_{min} = 544.268$ , where “null” denotes the absence of the upper limit for  $c$ .

$3/r_\Lambda^2(t) = 3/(c^2 t_\Lambda^2(t))$ , we investigate the  $\Lambda(t)$ CDM models corresponding to the length scales, including the Hubble horizon, the particle horizon and the future event horizon, and the time scales, including the age of the universe and the conformal time. It comes out that an accelerating expansion universe can be derived in the cases taking the Hubble horizon, the future event horizon, the age of the universe and the conformal time as the length scale or time scale. Furthermore, the modalities of the holographic dark energy and the agegraphic dark energy can be derived from the  $\Lambda(t)$ CDM models. In this scenario, the  $\Lambda(t)$ CDM model can be taken as the unified origin of the holographic and agegraphic dark energy models with interaction between the matter and the dark energy, where the interacting term is determined by  $Q = -\dot{\rho}_\Lambda$ .

However, in the last two cases, the contours are not closed for 68.3%, 95.4% and 99.7% confidence levels, which lead to the absence of the upper limit for the parameter  $c$ . We found that this problem chiefly originates from the evolution of  $\Omega_\Lambda$  with respect to redshift  $z$  with the variety of the  $c$ . In Figure 5, the evolutions of  $\Omega_\Lambda$  with respect to redshift  $z$  with the variety of  $c$  are displayed, corresponding to the three cases taking the future event horizon, the age of the universe and the conformal time as the length scale or time scale, where  $\Omega_{m0}$  takes the best-fit value for each case. The black solid line in each panel is plotted with the best-fit pair  $(\Omega_{m0}, c)$ . In the first panel of Figure 5, corresponding to take the future event horizon as the length scale, the best-fit value  $c = 0.72$  lies in the range where  $\Omega_\Lambda$  is sensitive to the variety of  $c$ , which makes the presence of the upper and lower limits for  $c$  in 68.3%, 95.4% and 99.7% confidence levels. In the second panel of Figure 5, corresponding to take the age of the universe as the time scale,  $\Omega_\Lambda$  is sensitive to the variety of  $c$  when  $c$  is small, however,  $\Omega_\Lambda$  is insensitive to the variety of  $c$  when  $c$  is big enough, further more, the best-fit value  $c = 23.3$  lies in the range where  $\Omega_\Lambda$  is insensitive to the variety of  $c$ . As a result, the lower limit of  $c$  exits, but one cannot work out its upper limit. In the third panel of Figure 5, corresponding to take the conformal time as the time scale, the situation is the same as the above one.

The left panel of Figure 6 displays the evolutions of  $w_\Lambda^{eff}$  with the best-fit  $c$  or  $(\Omega_{m0}, c)$ , corresponding to the four cases that can lead to the accelerating expansion universe. In the case of taking the Hubble horizon as length scale,  $w_\Lambda^{eff} = -0.587^{+0.025}_{-0.023}$  is quintessence like.  $w_\Lambda^{eff}$  crosses the phantom divide in the case of taking the future event horizon as a length scale. In the other two cases,  $w_\Lambda$  are big than  $-1$  all through and their variations with the variety of  $z$  are not significant. The right panel of Figure 6 shows the evolutions of  $q(z)$  with the corresponding best-fit  $c$  or  $(\Omega_{m0}, c)$  for the four cases. In the case of taking the Hubble horizon as length scale,  $q = -0.380^{+0.037}_{-0.034}$  is a constant. A problem deserves to be pointed out here. In this case, as it was discussed in [25], when  $c$  is a fixed constant, non-transition from decelerated expansion to accelerated expansion can be realized. The author of [25] proposed a

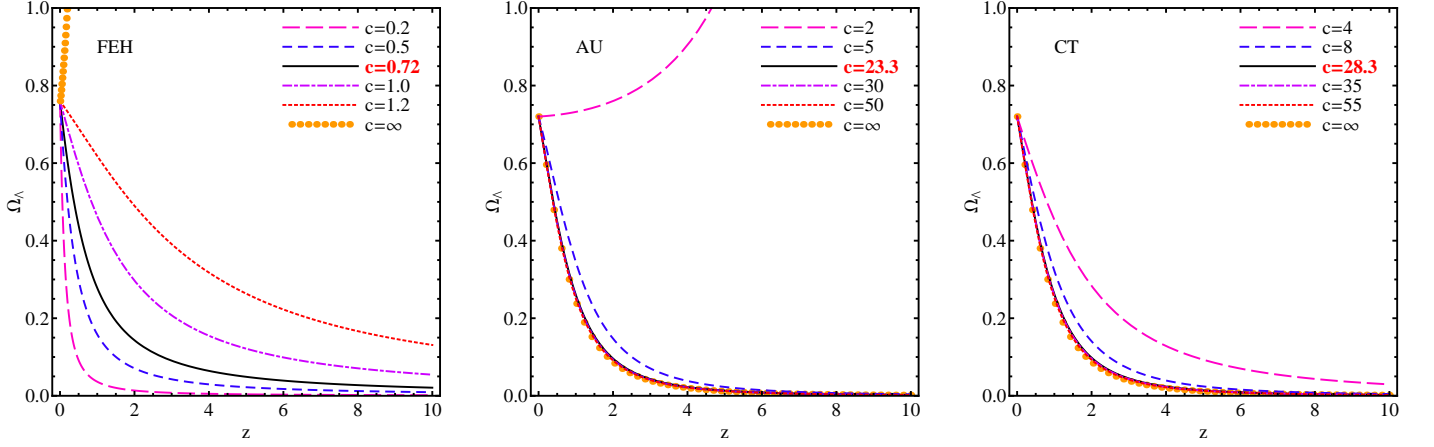


FIG. 5: The evolutions of  $\Omega_\Lambda$  with respect to redshift  $z$  with the variety of parameter  $c$ , corresponding to the future event horizon as a cosmological length scale (FEH for short), the age of the universe as a cosmological time scale (AU for short) and the conformal time as a cosmological time scale (CT for short), where  $\Omega_{m0}$  takes the best-fit value for each case. The black solid line in each panel is plotted with the best-fit pairs  $(\Omega_{m0}, c)$ .

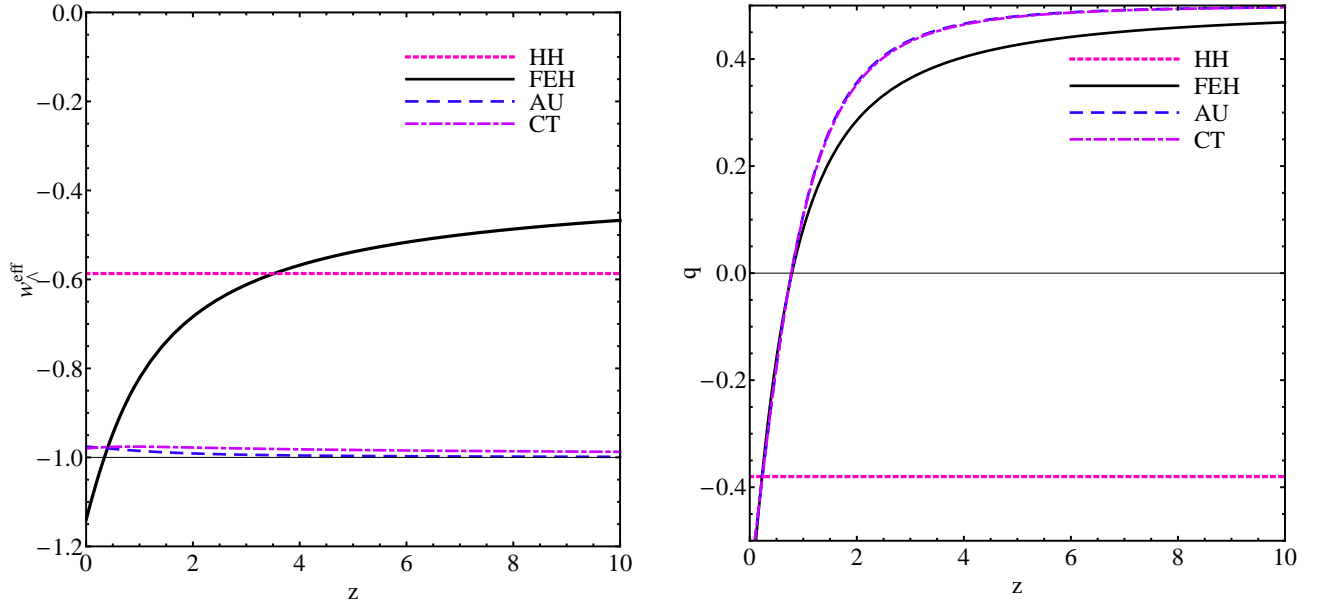


FIG. 6: The left panel displays the evolutions of  $w_\Lambda^{\text{eff}}$  with the corresponding best-fit  $c$  or  $(\Omega_{m0}, c)$ . The right panel shows the evolutions of  $q(z)$  with the corresponding best-fit  $c$  or  $(\Omega_{m0}, c)$ . HH, FEH, AU and CT are short for the Hubble horizon, the future event horizon, the age of the universe and the conformal time as a cosmological length scale or time scale, respectively.

possible remedy, that is to consider a time variable  $c$ , to solve this issue. Obviously, the evolutions of  $q(z)$  are very similar in the last three cases. As  $z$  grows,  $q$  goes to 0.5 at last, that is consistent with the  $\Lambda$ CDM model.

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